WELL-QUASI-ORDERING IN POSITROIDS

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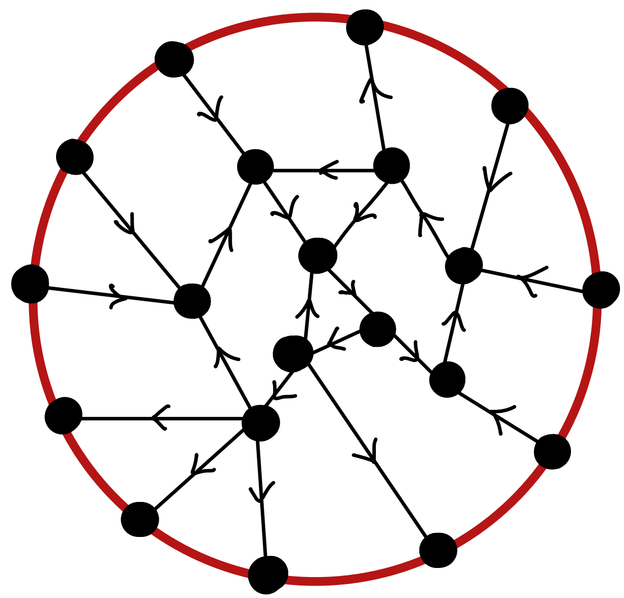
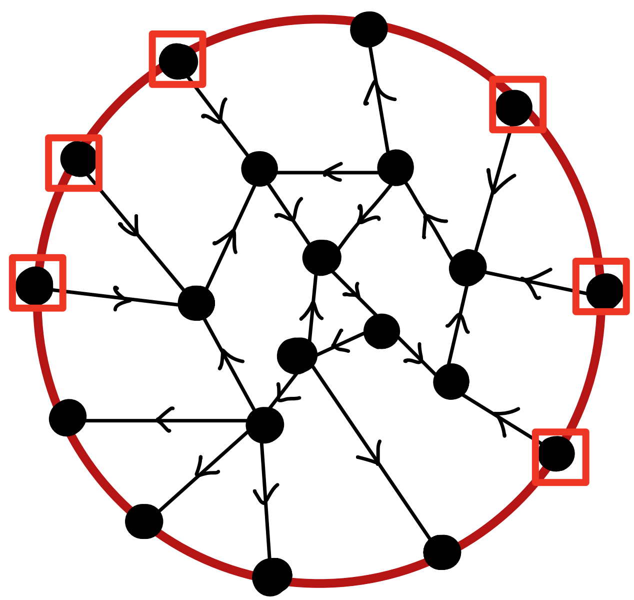
A matroid is a positroid if it can be represented by a real $r \times n$ matrix where every $r \times r$ submatrix has non-negative determinant.

For foundational material, see Total positivity, Grassmanians, and networks by Alexander Postnikov

Consider a planar directed graph drawn inside a disc, where every boundary vertex has degree one and every non-boundary vertex has in-degree or out-degree one.



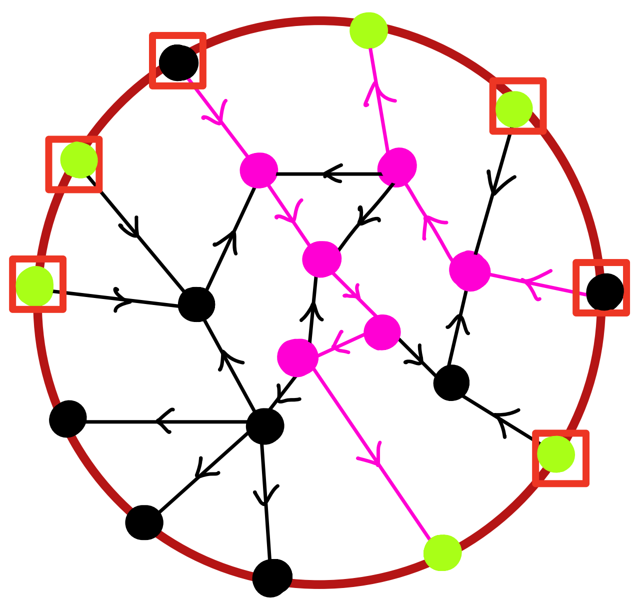
Let E be the set of boundary vertices.
Let T be the set of boundary vertices with out-degree one.



Define $\mathcal{B} = \{ B \subseteq E : T$ is linked to B by $|T|$ disjoint directed paths$\}$

$\mathcal{B}$ is a family of matroid bases. (E is the ground set.)

Positroids are exactly the matroids arising in this way.



Multipath matroids $\subseteq$ Positroids $\subseteq$ Gammoids

Theorem (Jose + Mayhew) If a minor-closed class of multi-path matroids has an infinite antichain then it contains $T_{r+1}(U_{r,r+1} \oplus U_{r,r+1})$ for infinitely many r.

Open Problem. Is the same statement is true for positroids?