WELL-QUASI-ORDERING
IN POSITROIDS

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A matroid is a positroid if it can be represented by a real $r \times n$ matrix where every $r \times r$ submatrix has non-negative determinant.

For foundational material, see "Total positivity, Grassmanians, and networks" by Alexander Postnikov.
Consider a planar directed graph drawn inside a disc, where every boundary vertex has degree one and every non-boundary vertex has in-degree or out-degree one.
Let $E$ be the set of boundary vertices. Let $T$ be the set of boundary vertices with out-degree one.
Define $\mathcal{B} = \{ B \in E : T \text{ is linked to } B \text{ by } |T| \text{ disjoint directed paths} \}$

$\mathcal{B}$ is a family of matroid bases. ($E$ is the ground set.) Positroids are exactly the matroids arising in this way.
Multipath matroids $\subseteq$ Positroids $\subseteq$ Gammoids

Theorem (Jose + Mayhew) If a minor-closed class of multipath matroids has an infinite antichain then it contains $Tr+1 (Ur,r+1 \oplus Ur,r+1)$ for infinitely many $r$.

Open Problem. Is the same statement is true for positroids?