Bounding the clique number

$\chi(G)$: the **chromatic number** of G

$\omega(G)$: the **clique number** of G

Q: What assumptions on G imply that $\chi(G) \leqslant f(\omega(G))$ for some f?

Such classes are called χ-**bounded**.

Direction: Geometric intersection graphs

Direction: Graphs admitting **decompositions**

Direction: H-**free** graphs for fixed H

Q: When can we choose f to be a polynomial?

Esperet: Is there a hereditary class of graphs that is χ-bounded, but **not** polynomially χ-bounded?

Bounding the biclique number

What if we exclude a **biclique** instead?

$\widehat{\omega}(G)$: largest t such that $K_{t,t}$ is a **subgraph** of G

Theorem (Kühn, Osthus)

For every graph H there is a function $f_H(\cdot)$ such that the following holds.

Suppose G does not contain **any subdivision** of H as an induced subgraph.

Then degeneracy$(G) \leqslant f_H(\widehat{\omega}(G))$.

Problem

Can we always choose $f_H(\cdot)$ to be a polynomial?

Works for: [BPRzTW 21, SSS 21$^+$]

paths, **trees**, **cycles**.

Next step: $K_{2,3}$?