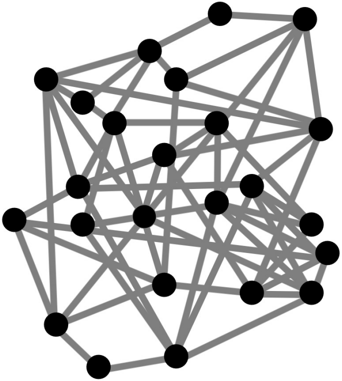
When is the maximum average degree of a graph tied to the size of its largest balanced biclique?



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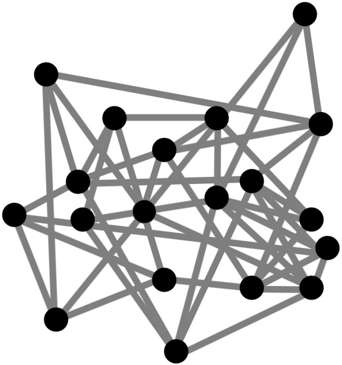

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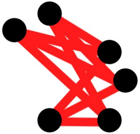

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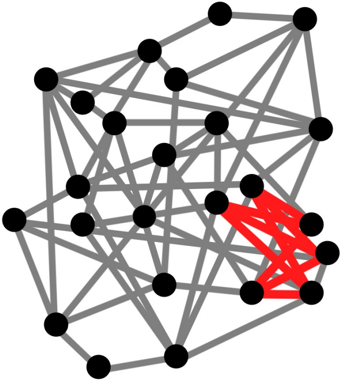
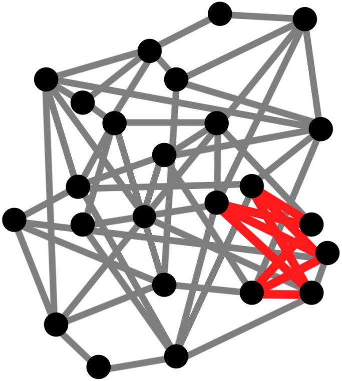


biclique number $\tau(G) :=$
maximum t so that G has $K_{t,t}$-subgraph

For which **hereditary** classes of graphs does there exist a function f so that $\text{avgdeg}(G) \leq f(\tau(G))$?



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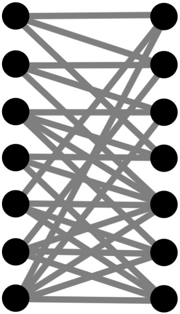

Theorem (Kwan, Letzter, Sudakov, Tran)

It is enough to consider the bipartite graphs in the class.

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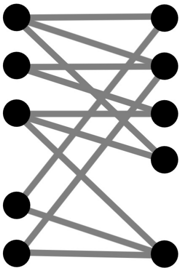

Theorem (McCarty, generalizes Kühn & Osthus)

This occurs iff there exists $c \in \mathbb{Z}$ such that every bipartite, 4-cycle-free graph in the class has avgdeg $\leq c$.

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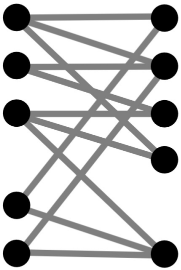

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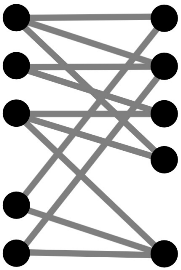

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Problem

Construct such a class where we cannot take $f = \mathbf{poly}(\tau)$.

It is unknown if such a class exists. See Esperet's Conjecture.