A <u>tangle</u> of order k in a graph G is an orientation $\mathcal{T}$ of the set of separations in G of order less than k, such that there are no $(A_1, B_1), (A_2, B_2), (A_3, B_3) \in \mathcal{T}$ with $G[A_1] \cup G[A_2] \cup G[A_3] = G$

Each $<k$-separation (A, B) of G induces a subset of $\mathrm{Tang}(G)$, namely $\{\mathcal{T} \mid (A,B) \in \mathcal{T}\}$.

The set of these subsets is denoted $\mathrm{Thicket}(G, k)$.

For G $(k-1)$-connected

1. $\emptyset$, $\mathrm{Tang}(G) \in \mathrm{Thicket}(G, k)$
2. For $\mathcal{T} \neq \mathcal{T}' \in \mathrm{Tang}(G,k)$ there is $X \in \mathrm{Tang}(G,k)$ with $|X \cap \{\mathcal{T}, \mathcal{T}'\}| = 1$
3. For $X, Y \in \mathrm{Thicket}(G,k)$, if $X \cap Y \neq \emptyset$ and $X \cup Y \neq \mathrm{Tang}(G,k)$ then $X \cap Y$, $X \cup Y \in \mathrm{Thicket}(G,k)$
4. $X \in \mathrm{Thicket}(G,k) \Rightarrow \overline{X} \in \mathrm{Thicket}(G,k)$

A <u>thicket</u> on a set E is a set Θ of subsets of E such that:

1. $\emptyset, E \in \Theta$
2. For $x \neq y \in E$ there is $X \in E$ with $|X \cap \{x,y\}| = 1$
3. For $X, Y \in \Theta$, if $X \cap Y \neq \emptyset$ and $X \cup Y \neq E$ then $X \cap Y, X \cup Y \in \Theta$
4. $X \in \Theta \Rightarrow \bar{X} \in \Theta$.

<u>Theorem</u> (Cunningham + Edmonds, Elm): For any thicket Θ on E there is a 2-connected graph G with

$$\Theta \cong \mathrm{Thicket}(G, 3)$$

A DIRECTED tangle of order k in a DIRECTED graph G is an orientation τ of the set of DIRECTED separations in G of order less than k, such that there are no $(A_1, B_1), (A_2, B_2), (A_3, B_3) \in \tau$ with $G[A_1] \cup G[A_2] \cup G[A_3] = G$

Each DIRECTED $<k$-separation (A,B) of G induces a subset of $DTang(G)$, namely $\{ \tau \mid (A,B) \in \tau \}$.

The set of these subsets is denoted $DThicket(G,k)$.

For G $(k-1)$-connected

① $\emptyset, DTang(G) \in DThicket(G,k)$

② For $\tau \neq \tau' \in DTang(G,k)$ there is $X \in DTang(G,k)$ with $|X \cap \{\tau, \tau'\}| = 1$

③ For $X, Y \in DThicket(G,k)$, if $X \cap Y \neq \emptyset$ and $X \cup Y \neq DTang(G,k)$

then $X \cap Y, X \cup Y \in DThicket(G,k)$

④ $\cancel{X \in DThicket(G,k) \Rightarrow \overline{X} \in DThicket(G,k)}$

A DIRECTED thicket on a set E is a set Θ of subsets of E such that:

1. $\emptyset, E \in \Theta$
2. For $x \neq y \in E$ there is $X \in E$ with $|X \cap \{x,y\}| = 1$
3. For $X, Y \in \Theta$, if $X \cap Y \neq \emptyset$ and $X \cup Y \neq E$ then $X \cap Y, X \cup Y \in \Theta$
4. ~~$X \in \Theta \Rightarrow \bar{X} \in \Theta$.~~

Question: Is there some $k \in \mathbb{N}$ such that for any DIRECTED thicket Θ on E there is a $(k-1)$-connected DIRECTED graph G with

$$\Theta \simeq \mathrm{DThicket}(G, k)$$