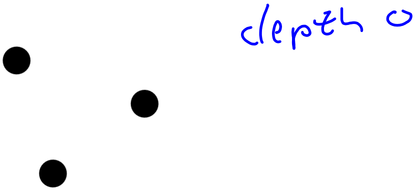
A class of graphs has **bounded shrub-depth** if every graph in it can be constructed by a bounded depth sequence, where

- $\mathrm{depth}(K_1) = 0$,
- $\mathrm{depth}(G_1 \uplus G_2) = \max(\mathrm{depth}(G_1), \mathrm{depth}(G_2))$, and
- for any $S \subseteq V(G)$, replacing $G[S]$ by its complement increases depth by 1.

Can we get an induced subgraph characterization?

A class of graphs has **bounded shrub-depth** if every graph in it can be constructed by a bounded depth sequence, where

- $\mathrm{depth}(K_1) = 0$,
- $\mathrm{depth}(G_1 \uplus G_2) = \max(\mathrm{depth}(G_1), \mathrm{depth}(G_2))$, and
- for any $S \subseteq V(G)$, replacing $G[S]$ by its complement increases depth by 1.



Can we get an induced subgraph characterization?

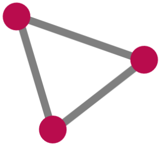
A class of graphs has **bounded shrub-depth** if every graph in it can be constructed by a bounded depth sequence, where

- $\mathrm{depth}(K_1) = 0$,
- $\mathrm{depth}(G_1 \uplus G_2) = \max(\mathrm{depth}(G_1), \mathrm{depth}(G_2))$, and
- for any $S \subseteq V(G)$, replacing $G[S]$ by its complement increases depth by 1.

Can we get an induced subgraph characterization?

A class of graphs has **bounded shrub-depth** if every graph in it can be constructed by a bounded depth sequence, where

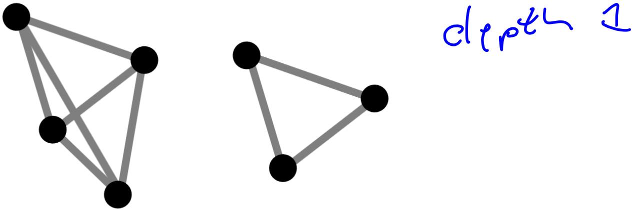
- $\mathrm{depth}(K_1) = 0$,
- $\mathrm{depth}(G_1 \uplus G_2) = \max(\mathrm{depth}(G_1), \mathrm{depth}(G_2))$, and
- for any $S \subseteq V(G)$, replacing $G[S]$ by its complement increases depth by 1.



Can we get an induced subgraph characterization?

A class of graphs has **bounded shrub-depth** if every graph in it can be constructed by a bounded depth sequence, where

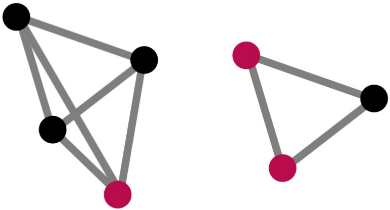
- $\mathrm{depth}(K_1) = 0$,
- $\mathrm{depth}(G_1 \uplus G_2) = \max(\mathrm{depth}(G_1), \mathrm{depth}(G_2))$, and
- for any $S \subseteq V(G)$, replacing $G[S]$ by its complement increases depth by 1.



Can we get an induced subgraph characterization?

A class of graphs has **bounded shrub-depth** if every graph in it can be constructed by a bounded depth sequence, where

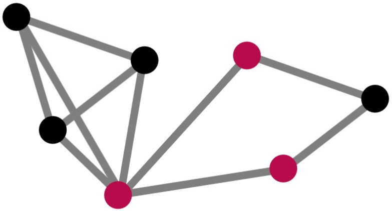
- $\mathrm{depth}(K_1) = 0$,
- $\mathrm{depth}(G_1 \uplus G_2) = \max(\mathrm{depth}(G_1), \mathrm{depth}(G_2))$, and
- for any $S \subseteq V(G)$, replacing $G[S]$ by its complement increases depth by 1.



Can we get an induced subgraph characterization?

A class of graphs has **bounded shrub-depth** if every graph in it can be constructed by a bounded depth sequence, where

- $\mathrm{depth}(K_1) = 0$,
- $\mathrm{depth}(G_1 \uplus G_2) = \max\left(\mathrm{depth}(G_1), \mathrm{depth}(G_2)\right)$, and
- for any $S \subseteq V(G)$, replacing $G[S]$ by its complement increases depth by 1.

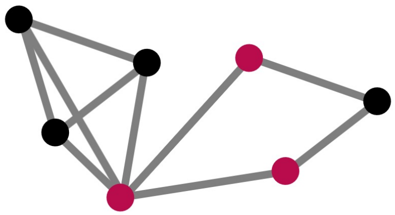


depth 2

Can we get an induced subgraph characterization?

A class of graphs has **bounded shrub-depth** if every graph in it can be constructed by a bounded depth sequence, where

- $\mathrm{depth}(K_1) = 0$,
- $\mathrm{depth}(G_1 \uplus G_2) = \max\left(\mathrm{depth}(G_1), \mathrm{depth}(G_2)\right)$, and
- for any $S \subseteq V(G)$, replacing $G[S]$ by its complement increases depth by 1.



Can we get an induced subgraph characterization?

Conjecture

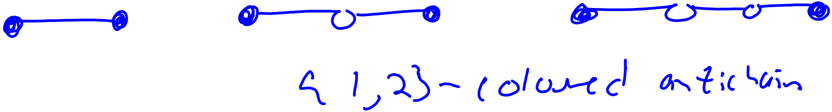
A hereditary class of graphs has **bounded shrub-depth** *if and only if it is ∞-WQO and forbids a semi-induced half-graph.*

- **∞-WQO:** For each n, there is no infinite antichain of $\{1, 2, \ldots, n\}$-coloured graphs under induced subgraphs where colours must match.
- **semi-induced half-graphs:** The following, induced between A and B.

Conjecture

A hereditary class of graphs has **bounded shrub-depth** *if and only if it is ∞-WQO and forbids a semi-induced half-graph.*

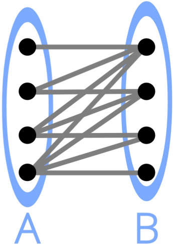
- **∞-WQO:** For each n, there is no infinite antichain of $\{1, 2, \ldots, n\}$-coloured graphs under induced subgraphs where colours must match.
- **semi-induced half-graphs:** The following, induced between A and B.



Conjecture

A hereditary class of graphs has **bounded shrub-depth** *if and only if it is* ∞*-WQO and forbids a semi-induced half-graph.*

- **∞-WQO:** For each n, there is no infinite antichain of $\{1, 2, \ldots, n\}$-coloured graphs under induced subgraphs where colours must match.
- **semi-induced half-graphs:** The following, induced between A and B.



- Characterization of bounded shrub-depth by vertex-minors (Kwon, M., Oum, and Wollan). [link]
- Related conjectures on 2-WQO (Daligault, Rao, Thomassé). [link]