

Extending bias to bicircuits

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Let $G = (V, E)$ be a graph. Let $k \geq 0$ be an integer. Then

$$\mathcal{C}_k = \{C \subseteq E : |C| = |V(C)| + k, C \text{ minimal}\}$$

forms the set of circuits of a matroid on E .

$$\mathcal{C}_0 = \{\text{cycles}\}, \quad \mathcal{C}_1 = \{\text{theta, tight/loose handcuffs}\} = \left\{ \begin{array}{c} \text{theta} \\ \text{tight handcuffs} \\ \text{loose handcuffs} \end{array} \right\}$$

$$\mathcal{C}_2 = \left\{ \begin{array}{c} \text{square with diagonal} \\ \text{theta with one edge} \\ \text{theta with two edges} \\ \text{theta with three edges} \\ \dots \\ \text{theta with four edges} \\ \text{theta with five edges} \\ \dots \end{array} \right\}$$

A **biased graph** is a pair (G, \mathcal{B}_0) where $\mathcal{B}_0 \subseteq \mathcal{C}_0$ satisfies:

$$\text{if } C_1, C_2 \in \mathcal{B}_0 \text{ form a theta, then } C_1 \triangle C_2 \in \mathcal{B}_0. \quad (*)$$

There is a natural matroid $M_0(G, \mathcal{B}_0)$ associated with \mathcal{B}_0 such that

- ▶ $M_0(G, \mathcal{C}_0)$ has circuits \mathcal{C}_0 , and
- ▶ $M_0(G, \emptyset)$ has circuits \mathcal{C}_1 .

Question: Is there an analogous condition to $(*)$ for subsets $\mathcal{B}_1 \subseteq \mathcal{C}_1$ giving an associated matroid $M_1(G, \mathcal{B}_1)$ such that

- ▶ $M_1(G, \mathcal{C}_1)$ has circuits \mathcal{C}_1 , and
- ▶ $M_1(G, \emptyset)$ has circuits \mathcal{C}_2 ?