Extending bias to bicircuits

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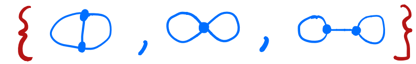
Let $G = (V, E)$ be a graph. Let $k \geq 0$ be an integer. Then

$$\mathcal{C}_k = \{C \subseteq E : |C| = |V(C)| + k, C \text{ minimal}\}$$

forms the set of circuits of a matroid on E.

$\mathcal{C}_0 = \{\text{cycles}\}$, $\quad \mathcal{C}_1 = \{\text{theta, tight/loose handcuffs}\}$ = 

$\mathcal{C}_2 =$ 

A **biased graph** is a pair $(G, \mathcal{B}_0)$ where $\mathcal{B}_0 \subseteq \mathcal{C}_0$ satisfies:

$$\text{if } C_1, C_2 \in \mathcal{B}_0 \text{ form a theta, then } C_1 \triangle C_2 \in \mathcal{B}_0. \qquad (*)$$

There is a natural matroid $M_0(G, \mathcal{B}_0)$ associated with $\mathcal{B}_0$ such that

- $M_0(G, \mathcal{C}_0)$ has circuits $\mathcal{C}_0$, and
- $M_0(G, \emptyset)$ has circuits $\mathcal{C}_1$.

Question: Is there an analogous condition to $(*)$ for subsets $\mathcal{B}_1 \subseteq \mathcal{C}_1$ giving an associated matroid $M_1(G, \mathcal{B}_1)$ such that

- $M_1(G, \mathcal{C}_1)$ has circuits $\mathcal{C}_1$, and
- $M_1(G, \emptyset)$ has circuits $\mathcal{C}_2$?