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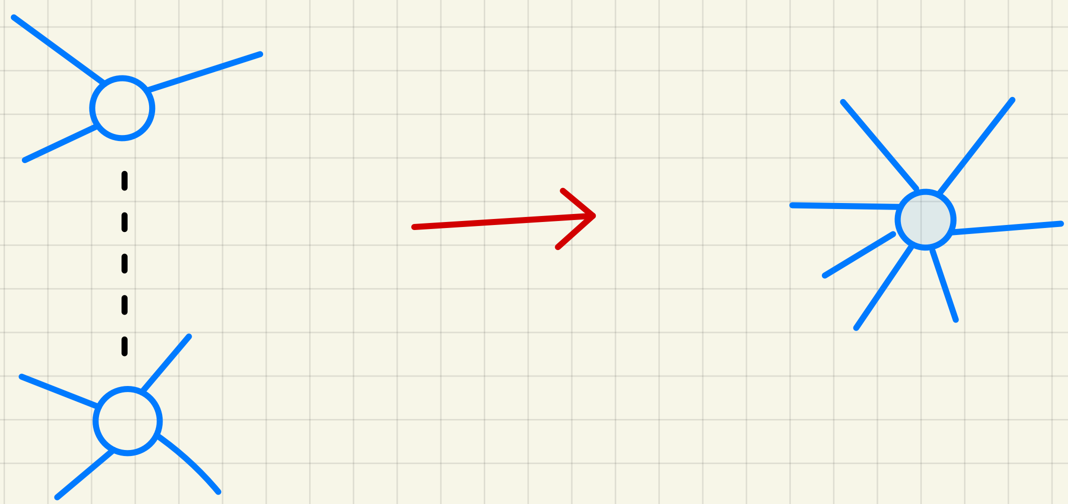
Questions about graphs

Weaker Conjecture

If $\chi(G) \geq f(k,g)$, then G is the image, under homomorphism of a graph with girth $\geq g$ and chromatic number k.



Wild Conjecture:

If $\text{girth}(H) \gg |G|$ and $\chi(G) \gg \chi(H)$, then $H \xrightarrow{\text{hom}} G$.

Question: If $\chi(G) \geq 1000$, and H is a cubic graph with $\text{girth} \geq f(G)$, is there a homomorphism from H into G?

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Questions about matroids.

Conjecture: If M is a binary matroid with $\chi(M) \geq f(k)$, then M has a Δ-free restriction with critical number k.

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