

What is the q -analogue of a graph?

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What is the q -analogue of a graph?

Its cycles should be the circuits of a q -matroid:

q -matroid: a pair (E, \mathcal{C}) with

- ▶ $E = \mathbb{F}_q^n$ finite dimensional vector space;
- ▶ \mathcal{C} family of subspaces of E , the *circuits*, with
 - (C1) $0 \notin \mathcal{C}$.
 - (C2) If $C_1, C_2 \in \mathcal{C}$ and $C_1 \subseteq C_2$, then $C_1 = C_2$.
 - (C3) For all $C_1, C_2 \in \mathcal{C}$ distinct and $X \subseteq E$ of codimension 1, there is a $C_3 \in \mathcal{C}$ such that $C_3 \subseteq (C_1 + C_2) \cap X$.

What is the q -analogue of a graph?

Wild speculation:

graph	\rightsquigarrow	hypergraph
$k + 1$ vertices	\rightsquigarrow	all 1-dim spaces of \mathbb{F}_q^{k+1} i.e., points in $\text{PG}(k, q)$
n edges	\rightsquigarrow	hyperedges coming from 2-dim spaces i.e., lines in $\text{PG}(k, q)$
cycles	\rightsquigarrow	???