Problem Session — 14 December 2020

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Definition

Let $\mathcal{A}$ be a class of matroids.

$\mathcal{A}$ is an antichain if for every $M, N \in \mathcal{A}$, there is no minor of M isomorphic to N.

Observation

Infinite antichains exist.

Corollary

There are uncountably many minor-closed classes of matroids.

Definition

A minor-closed class of matroids is well-quasi-ordered if it does not contain any infinite antichain.

Definition

A matroid is nested if for every pair Z, Z' of cyclic flats, either $Z \subseteq Z'$ or $Z' \subseteq Z$.

Example

The class of nested matroids is minor-closed and well-quasi-ordered.

It has infinitely many excluded minors: the truncation of the direct sum $U_{r,r+1} \oplus U_{r,r+1}$ to rank r is an excluded minor.

Conjecture

There are only countably many minor-closed classes that are well-quasi-ordered.

If there are uncountably many classes of graphs that are closed under induced subgraphs and well-quasi-ordered under the induced subgraph relation, then this conjecture fails.